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1. [**5**] Evaluate $2 + 5 + 8 + \cdots + 101$.

Answer: _____

2. [**5**] Two fair six-sided dice are rolled. What is the probability that their sum is at least 10?

Answer: _____

3. [**5**] A square is inscribed in a circle of radius 1. Find the perimeter of the square.

Answer: _____

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4. **[6]** Find the minimum possible value of $(x^2 + 6x + 2)^2$ over all real numbers x.

Answer: _____

5. **[6]** How many positive integers less than 100 are relatively prime to 200? (Two numbers are *relatively prime* if their greatest common factor is 1.)

Answer: _____

6. **[6]** A right triangle has area 5 and a hypotenuse of length 5. Find its perimeter.

Answer: _____

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7. [**7**] How many 9-digit numbers are there with unique digits from 1 to 9 such that the first five digits form an increasing series and the last five digits form a decreasing series?

Answer: _____

8. [**7**] Compute the number of ordered triples (a, b, c) with $0 \leq a, b, c \leq 30$ such that 73 divides $8^a + 8^b + 8^c$.

Answer: _____

9. [**7**] Mark plays a game with a circle that has six spaces around it, labeled 1 through 6, and a marker. The marker starts on space 1. On each move, Mark flips a coin. If he gets tails, the marker stays where it is, and if he gets heads, he then rolls a die, with numbers 1 through 6, and moves the marker forward the number of spaces that is rolled without stopping (if the marker passes space 6, it will keep going to space 1). What is the expected numbers of moves for the marker to stop on space 6 for the first time?

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10. [8] Compute the number of ordered pairs (a, b) of positive integers such that a and b divide 5040 but share no common factors greater than 1.

Answer: _____

11. [8] Kylie is trying to count to 202250. However, this would take way too long, so she decides to only write down positive integers from 1 to 202250, inclusive, that are divisible by 125. How many times does she write down the digit 2?

Answer: _____

12. [8] Let circles C_1 and C_2 be internally tangent at point P, with C_1 being the smaller circle. Consider a line passing through P which intersects C_1 at Q and C_2 at R. Let the line tangent to C_2 at R and the line perpendicular to $\overline{PR}$ passing through Q intersect at a point S outside both circles. Given that $SR = 5$, $RQ = 3$, and $QP = 2$, compute the radius of C_2.

Answer: _____

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Instructions: You will recieve three problems in each set. Problems in later sets will have higher point values - as denoted in brackets (i.e. [10]). Once you solve all problems in a set, you can turn in your slip and recieve the next set of problems. Print your team name in the space provided, and write your answer legibly in the answer blank next to the corresponding problem. Incorrect or blank answers are worth 0 points, so you can skip to the next set if you are stuck. Answers must be expressed in simplest terms, and are not neccesarily integers.

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13. **[9]** Real numbers x and y satisfy the system of equations

$$x^3 + 3x^2 = -3y - 1$$
$$y^3 + 3y^2 = -3x - 1.$$

What is the greatest possible value of x?

Answer: _____

14. **[9]** Isaac writes each fraction $\frac{1^2}{300}, \frac{2^2}{300}, \ldots, \frac{300^2}{300}$ in reduced form. Compute the sum of all denominators over all the reduced fractions that Isaac writes down (the denominator may be 1).

Answer: _____

15. **[9]** Let $f(x)$ be a function acting on a string of 0s and 1s, defined to be the number of substrings of x that have at least one 1, where a substring is a contiguous sequence of characters in x. Let S be the set of binary strings with 24 ones and 100 total digits. Compute the maximum possible value of $f(s)$ over all $s \in S$.

For example, $f(110) = 5$ as $\underline{1}10$, $1\underline{1}0$, $\underline{11}0$, $1\underline{10}$, and $\underline{110}$ are all substrings including a 1. Note that $11\underline{0}$ is not such a substring.

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16. **[10]** When not writing power rounds, Eric likes to climb trees. The strength in his arms as a function of time is $s(t) = t^3 - 3t^2$. His climbing velocity as a function of the strength in his arms is $v(s) = s^5 + 9s^4 + 19s^3 - 9s^2 - 20s$. At how many (possibly negative) points in time is Eric stationary?

Answer: _____

17. **[10]** Consider a triangle $\triangle ABC$ with angles $\angle ACB = 60^\circ$, $\angle ABC = 45^\circ$. The circumcircle around $\triangle ABH$, where H is the orthocenter of $\triangle ABC$, intersects BC for a second time in point P, and the center of that circumcircle is O_c. The line PH intersects AC in point Q, and N is center of the circumcircle around $\triangle AQP$. Find $\angle NO_cP$.

Answer: _____

18. **[10]** If x, y are positive real numbers and $xy^3 = \frac{16}{9}$, what is the minimum possible value of $3x + y$?

Answer: _____

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Answer: _____

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19. **[11]** Shelly writes down a vector $v = (a, b, c, d)$, where $0 < a < b < c < d$ are integers. Let $\sigma(v)$ denote the set of 24 vectors whose coordinates are a, b, c, and d in some order. For instance, $\sigma(v)$ contains (b, c, d, a). Shelly notes that there are 3 vectors in $\sigma(v)$ whose sum is of the form (s, s, s, s) for some s. What is the smallest possible value of d?

Answer: _____

20. **[11]** A positive integer is called *jubilant* if the number of 1's in its binary representation is even. For example, $6 = 110_2$ is a jubilant number. What is the 2009th smallest jubilant number?

Answer: _____

21. **[11]** Simplify

$$2\cos^2(\ln(2009)i) + i\sin(\ln(4036081)i).$$

Answer: _____

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Instructions: You will recieve three problems in each set. Problems in later sets will have higher point values - as denoted in brackets (i.e. [12]). Once you solve all problems in a set, you can turn in your slip and recieve the next set of problems. Print your team name in the space provided, and write your answer legibly in the answer blank next to the corresponding problem. Incorrect or blank answers are worth 0 points, so you can skip to the next set if you are stuck. Answers must be expressed in simplest terms, and are not neccesarily integers.

22. **[12]** Kelvin the Frog and 10 of his relatives are at a party. Every pair of frogs is either *friendly* or *unfriendly*. When 3 pairwise friendly frogs meet up, they will gossip about one another and end up in a *fight* (but stay *friendly* anyway). When 3 pairwise unfriendly frogs meet up, they will also end up in a *fight*. In all other cases, common ground is found and there is no fight. If all $\binom{11}{3}$ triples of frogs meet up exactly once, what is the minimum possible number of fights?

Answer: _____

23. **[12]** Five points are chosen uniformly at random on a segment of length 1. What is the expected distance between the closest pair of points?

Answer: _____

24. **[12]** At a recent math contest, Evan was asked to find $2^{2016} \pmod p$ for a given prime number p with $100 < p < 500$. Evan has forgotten what the prime p was, but still remembers how he solved it:

- Evan first tried taking 2016 modulo $p-1$, but got a value e larger than 100.
- However, Evan noted that $e - \frac{1}{2}(p-1) = 21$, and then realized the answer was $-2^{21} \pmod p$.

What was the prime p?

Answer: _____

Set 8

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23. **[12]** Five points are chosen uniformly at random on a segment of length 1. What is the expected distance between the closest pair of points?

Answer: _____

24. **[12]** At a recent math contest, Evan was asked to find $2^{2016} \pmod p$ for a given prime number p with $100 < p < 500$. Evan has forgotten what the prime p was, but still remembers how he solved it:

- Evan first tried taking 2016 modulo $p-1$, but got a value e larger than 100.
- However, Evan noted that $e - \frac{1}{2}(p-1) = 21$, and then realized the answer was $-2^{21} \pmod p$.

What was the prime p?

Answer: _____

Set 8

Team Name: _____

Instructions: You will recieve three problems in each set. Problems in later sets will have higher point values - as denoted in brackets (i.e. [12]). Once you solve all problems in a set, you can turn in your slip and recieve the next set of problems. Print your team name in the space provided, and write your answer legibly in the answer blank next to the corresponding problem. Incorrect or blank answers are worth 0 points, so you can skip to the next set if you are stuck. Answers must be expressed in simplest terms, and are not neccesarily integers.

22. **[12]** Kelvin the Frog and 10 of his relatives are at a party. Every pair of frogs is either *friendly* or *unfriendly*. When 3 pairwise friendly frogs meet up, they will gossip about one another and end up in a *fight* (but stay *friendly* anyway). When 3 pairwise unfriendly frogs meet up, they will also end up in a *fight*. In all other cases, common ground is found and there is no fight. If all $\binom{11}{3}$ triples of frogs meet up exactly once, what is the minimum possible number of fights?

Answer: _____

23. **[12]** Five points are chosen uniformly at random on a segment of length 1. What is the expected distance between the closest pair of points?

Answer: _____

24. **[12]** At a recent math contest, Evan was asked to find $2^{2016} \pmod p$ for a given prime number p with $100 < p < 500$. Evan has forgotten what the prime p was, but still remembers how he solved it:

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Set 9

Team Name: _____

Instructions: You will recieve three problems in each set. Problems in later sets will have higher point values - as denoted in brackets (i.e. [13]). Once you solve all problems in a set, you can turn in your slip and recieve the next set of problems. Print your team name in the space provided, and write your answer legibly in the answer blank next to the corresponding problem. Incorrect or blank answers are worth 0 points, so you can skip to the next set if you are stuck. Answers must be expressed in simplest terms, and are not neccesarily integers.

25. **[13]** What is the smallest positive integer n which cannot be written in any of the following forms?

- $n = 1 + 2 + \cdots + k$ for a positive integer k.
- $n = p^k$ for a prime number p and integer k.
- $n = p + 1$ for a prime number p.
- $n = pq$ for some distinct prime numbers p and q.

Answer: _____

26. **[13]** Consider a permutation $(a_1, a_2, a_3, a_4, a_5)$ of $\{1, 2, 3, 4, 5\}$. We say the tuple $(a_1, a_2, a_3, a_4, a_5)$ is *flawless* if for all $1 \leq i < j < k \leq 5$, the sequence (a_i, a_j, a_k) is *not* an arithmetic progression (in that order). Find the number of flawless 5-tuples.

Answer: _____

27. **[13]** In triangle ABC, let the parabola with focus A and directrix BC intersect sides AB and AC at A_1 and A_2, respectively. Similarly, let the parabola with focus B and directrix CA intersect sides BC and BA at B_1 and B_2, respectively. Finally, let the parabola with focus C and directrix AB intersect sides CA and CB at C_1 and C_2, respectively.
If triangle ABC has sides of length 5, 12, and 13, find the area of the triangle determined by lines A_1C_2, B_1A_2 and C_1B_2.

Answer: _____

Set 9

Team Name: _____

Instructions: You will recieve three problems in each set. Problems in later sets will have higher point values - as denoted in brackets (i.e. [13]). Once you solve all problems in a set, you can turn in your slip and recieve the next set of problems. Print your team name in the space provided, and write your answer legibly in the answer blank next to the corresponding problem. Incorrect or blank answers are worth 0 points, so you can skip to the next set if you are stuck. Answers must be expressed in simplest terms, and are not neccesarily integers.

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